Suggested solutions to the Contract Theory exam on Jan. 21, 2011
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Question 1[1]

Consider the following extension of the basic adverse selection model. A firm (the agent) interacts with a government procurement agency (the principal). The firm produces office material that the procurement agency wants to buy. The firm's cost of producing q units of office material is given by the function $C(q,\theta)$, where θ is an efficiency parameter. This function satisfies

$$C(0,\theta)=0, \qquad C_q>0, \qquad C_{qq}\geq 0, \qquad C_\theta>0, \qquad C_{q\theta}>0, \qquad C_{qq\theta}\geq 0.$$

The value for the procurement agency of receiving q units of office material is given by the function $S(q)$, which satisfies

$$S'(q)>0, \qquad S''(q)<0, \qquad S(0)=0.$$

The efficiency parameter θ can take two values: $\theta \in \{\underline{\theta},\overline{\theta}\}$, with $0<\underline{\theta}<\overline{\theta}$. Initially (and this is where the model differs from the one we studied in the course), neither the firm nor the procurement agency knows the value of θ: they both believe that

$$\Pr[\theta=\underline{\theta}]=\nu \qquad \text{and} \qquad \Pr[\theta=\overline{\theta}]=1-\nu,$$

with $0<\nu<1$. However, the firm can, if incurring a cost $\gamma>0$, learn the value of θ. The timing of events is as follows.

1. **The procurement agency chooses a menu of contracts. A contract can specify the quantity q that the firm must produce and deliver and the payment t that the firm will receive.**

2. **The firm decides whether or not to incur information gathering costs γ to learn θ. The procurement agency cannot observe whether the firm incurs γ, nor can it observe the value of θ that the firm possibly learns.**

3. **The firm decides whether to reject all contracts in the menu or to accept one of them.**

[1]By accident, given the way the question is phrased it is not possible to answer the b) part of this problem completely. In particular, it is possible to show that $\underline{q}^{SB}\geq \underline{q}^{FB}$. But to determine whether $\underline{q}^{SB}=\underline{q}^{FB}$ or $\underline{q}^{SB}>\underline{q}^{FB}$, one has to make more specific assumptions about the model. For example, in these solutions I show that we must have $\underline{q}^{SB}=\underline{q}^{FB}$ if γ is strictly positive but sufficiently small. This issue did not affect the fairness of the grading — the grading was generous on this question as the question was relatively hard. Still, I would never deliberately ask a question that cannot be answered given the information that is provided.

Overall, the good thing with this exam question is that it is fairly easy for any student who has studied a bit to derive at least some results. At the same time it is sufficiently challenging for the most able and ambitious students (although the very last bit was more challenging than intended).

4. **If the firm accepted a contract at date 3, production takes place and the procurement agency pays the firm the contractually specified payment t.**

The procurement agency is risk neutral and its payoff, given a quantity q and a payment t, equals $V = S(q) - t$. The firm is also risk neutral and its payoff, given a quantity q and a payment t, equals $U = t - C(q,\theta) - \gamma$ if it has gathered information at date 2 and $U = t - C(q,\theta)$ otherwise. If the firm rejects all contracts at date 3, then its payoff equals $-\gamma$ if it has gathered information at date 2 and zero otherwise.

a) **Suppose the procurement agency wants to induce the firm to gather information. Also suppose that the parameters of the model are such that it is optimal to interact with both types and to offer them distinct contracts. Then we can write the procurement agency's problem as follows. The principal chooses $\left(\underline{q}, \overline{q}, \underline{t}, \overline{t}\right)$ so as to maximize its expected payoff,**

$$V\left(\underline{t}, \underline{q}, \overline{t}, \overline{q}\right) = \nu\left[S\left(\underline{q}\right) - \underline{t}\right] + (1-\nu)\left[S\left(\overline{q}\right) - \overline{t}\right],$$

subject to six constraints:

$$\overline{t} - C\left(\overline{q}, \overline{\theta}\right) \geq 0, \qquad \text{(IR-bad)}$$

$$\underline{t} - C\left(\underline{q}, \underline{\theta}\right) \geq 0, \qquad \text{(IR-good)}$$

$$\overline{t} - C\left(\overline{q}, \overline{\theta}\right) \geq \underline{t} - C\left(\underline{q}, \overline{\theta}\right), \qquad \text{(IC-bad)}$$

$$\underline{t} - C\left(\underline{q}, \underline{\theta}\right) \geq \overline{t} - C\left(\overline{q}, \underline{\theta}\right), \qquad \text{(IC-good)}$$

$$\begin{aligned} & \nu\left[\underline{t} - C\left(\underline{q}, \underline{\theta}\right)\right] + (1-\nu)\left[\overline{t} - C\left(\overline{q}, \overline{\theta}\right)\right] - \gamma \\ \geq \; & \underline{t} - \nu C\left(\underline{q}, \underline{\theta}\right) - (1-\nu) C\left(\underline{q}, \overline{\theta}\right), \end{aligned} \qquad \text{(IG-good)}$$

$$\begin{aligned} & \nu\left[\underline{t} - C\left(\underline{q}, \underline{\theta}\right)\right] + (1-\nu)\left[\overline{t} - C\left(\overline{q}, \overline{\theta}\right)\right] - \gamma \\ \geq \; & \overline{t} - \nu C\left(\overline{q}, \underline{\theta}\right) - (1-\nu) C\left(\overline{q}, \overline{\theta}\right). \end{aligned} \qquad \text{(IG-bad)}$$

Explain (briefly) in words what each one of the six constraints says.

- We have supposed that the procurement agency wants to induce the firm to indeed incur the information acquisition cost, which means that the firm will know its type at the point in time when it chooses which contract (if any) to pick from the menu.
- The constraint IR-bad is the bad type's individual rationality (or participation) constraint. It ensures that the bad type wants to participate. It says that the firm, when having learned that it's the bad type, prefers the bad type's contract to rejecting all contracts in the menu (which yields the outside option payoff zero).

- The constraint IR-good is the good type's individual rationality (or participation) constraint. It ensures that the good type wants to participate. It says that the firm, when having learned that it's the good type, prefers the good type's contract to rejecting all contracts in the menu (which yields the outside option payoff zero).
- The constraint IC-bad is the bad type's incentive compatibility constraint. It says that the firm, when having learned that it's the bad type, prefers the bad type's contract to the good type's contract.
- The constraint IC-good is the good type's incentive compatibility constraint. It says that the firm, when having learned that it's the good type, prefers the good type's contract to the bad type's contract.
- The constraints IG-bad and IG-good are the ones that ensure that the firm has an incentive to gather information.
  - The left-hand side of each of these constraints is the firm's expected payoff at the stage of information acquisition, given that it indeed acquires information and then chooses the contract that the procurement agency wants it to choose (this is the best the firm can do if having acquired information, given that the IR and IC constraints are not violated).
  - The right-hand side of IG-bad is the expected payoff the firm would get if not acquiring information (thus not learning its type) and then picking the contract meant for the bad type; the right-hand side of IG-good is the same, but for the case where the firm picks the contract meant for the good type. If the firm did not acquire information, then it would pick either the bad or the good type's contract, depending on which one gave the highest expected payoff (at least one of these options must be better than the outside option payoff of zero). Therefore both IG-bad and IG-good must hold to ensure that the firm wants to acquire information.

b) **Let the first-best quantities, $\overline{q}^{FB}$ and $\underline{q}^{FB}$, be defined in the usual way by $S'\left(\overline{q}^{FB}\right) = C_q\left(\overline{q}^{FB}, \overline{\theta}\right)$ and $S'\left(\underline{q}^{FB}\right) = C_q\left(\underline{q}^{FB}, \underline{\theta}\right)$. Let the second-best quantities, $\overline{q}^{SB}$ and $\underline{q}^{SB}$, be the ones that solve the above problem. Show (by solving as much as you need of the problem) how $\overline{q}^{SB}$ relates to $\overline{q}^{FB}$, and how $\underline{q}^{SB}$ relates to $\underline{q}^{FB}$. You are allowed to assume that the second-order condition is satisfied (and you will not get any credit if you nevertheless investigate that.)**

- **Hint: Is (IC-bad) implied by (IG-good)? Is (IC-good) implied by (IG-bad)?**
- Before solving the problem, it's useful to try to simplify it by eliminating some of the constraints.
- The first hint suggests that IG-good implies IC-bad. Indeed, we can rewrite IG-good as

$$(1-\nu)\left[\overline{t} - \underline{t} + C\left(\underline{q}, \overline{\theta}\right) - C\left(\overline{q}, \overline{\theta}\right)\right] \geq \gamma.$$

Both $(1-\nu)$ and γ are strictly positive. Therefore the expression in square brackets must be non-negative for the inequality to hold. But the expression in square brackets being non-negative is exactly the IC-bad constraint. This proves that IG-good implies IC-bad.

- The second hint suggests that IG-bad implies IC-good. We can rewrite IG-bad as

$$\nu\left[\underline{t}-\overline{t}+C\left(\overline{q},\underline{\theta}\right)-C\left(\underline{q},\underline{\theta}\right)\right]\geq\gamma.$$

Both ν and γ are strictly positive. Therefore the expression in square brackets must be non-negative for the inequality to hold. But the expression in square brackets being non-negative is exactly the IC-good constraint. This proves that IG-bad implies IC-good.

- Finally, we can make use of the fact that here, as in our standard model, IC-good and IR-bad jointly imply IR-good. Proof:

$$\underline{t}-C\left(\underline{q},\underline{\theta}\right)\overset{\text{(i)}}{\geq}\overline{t}-C\left(\overline{q},\underline{\theta}\right)\overset{\text{(ii)}}{\geq}\overline{t}-C\left(\overline{q},\overline{\theta}\right)\overset{\text{(iii)}}{\geq}0.$$

Here inequality (i) follows from IC-good. Inequality (ii) follows from the fact that the cost function is increasing in θ and $\overline{\theta}>\underline{\theta}$ (i.e., a bad type produces a given quantity at a higher cost). Inequality (iii) follows from IR-bad. The above series of inequalities says in particular that $\underline{t}-C\left(\underline{q},\underline{\theta}\right)\geq 0$, which is IR-good. Hence we have proven that IC-good and IR-bad jointly imply IR-good.

- The above results mean that we safely can ignore IC-bad, IC-good, and IR-good, as they are implied by the other constraints.
- The Lagrangian associated with the remaining problem is:

$$\begin{aligned}\mathcal{L} &= \nu\left[S\left(\underline{q}\right)-\underline{t}\right]+(1-\nu)\left[S\left(\overline{q}\right)-\overline{t}\right]+\lambda\left[\overline{t}-C\left(\overline{q},\overline{\theta}\right)\right]\\ &\quad+\underline{\mu}\left[(1-\nu)\left[\overline{t}-\underline{t}+C\left(\underline{q},\overline{\theta}\right)-C\left(\overline{q},\overline{\theta}\right)\right]-\gamma\right]\\ &\quad+\overline{\mu}\left[\nu\left[\underline{t}-\overline{t}+C\left(\overline{q},\underline{\theta}\right)-C\left(\underline{q},\underline{\theta}\right)\right]-\gamma\right],\end{aligned}$$

where λ (≥ 0) is the shadow price for IR-bad, $\underline{\mu}$ (≥ 0) is the shadow price for IG-good, and $\overline{\mu}$ (≥ 0) is the shadow price for IG-bad.

- Differentiating the Lagrangian with respect to the choice variables $\underline{t}$, $\overline{t}$, $\underline{q}$, and $\overline{q}$, and then setting the resulting expression equal to zero, yields the following first-order conditions.
- FOC w.r.t. $\underline{t}$:

$$\frac{\partial\mathcal{L}}{\partial\underline{t}}=-\nu-\underline{\mu}(1-\nu)+\overline{\mu}\nu=0. \tag{1}$$

- FOC w.r.t. $\overline{t}$:

$$\frac{\partial\mathcal{L}}{\partial\overline{t}}=-(1-\nu)+\lambda+\underline{\mu}(1-\nu)-\overline{\mu}\nu=0. \tag{2}$$

- FOC w.r.t. $\underline{q}$:

$$\frac{\partial\mathcal{L}}{\partial\underline{q}}=\nu S'\left(\underline{q}\right)+\underline{\mu}(1-\nu)C_q\left(\underline{q},\overline{\theta}\right)-\overline{\mu}\nu C_q\left(\underline{q},\underline{\theta}\right)=0. \tag{3}$$

- FOC w.r.t. $\overline{q}$:

$$\frac{\partial \mathcal{L}}{\partial \overline{q}} = (1-\nu) S'(\overline{q}) - \lambda C_q\left(\overline{q}, \overline{\theta}\right) - \underline{\mu}(1-\nu) C_q\left(\overline{q}, \overline{\theta}\right) + \overline{\mu}\nu C_q\left(\overline{q}, \underline{\theta}\right) = 0. \tag{4}$$

- Adding (1) and (2) yields

$$\lambda = 1. \tag{5}$$

The fact that $\lambda > 0$ implies that IR-bad binds.

- We can rewrite (1) as

$$\overline{\mu}\nu = \underline{\mu}(1-\nu) + \nu. \tag{6}$$

Since $\nu \in (0,1)$ and $\underline{\mu} \geq 0$, this implies that $\overline{\mu} > 0$; hence, IG-bad binds.

- Rewrite (4) using (5) and (6):

$$\begin{aligned}
(1-\nu) S'(\overline{q}) &= \lambda C_q\left(\overline{q}, \overline{\theta}\right) + \underline{\mu}(1-\nu) C_q\left(\overline{q}, \overline{\theta}\right) - \overline{\mu}\nu C_q\left(\overline{q}, \underline{\theta}\right) \\
&= C_q\left(\overline{q}, \overline{\theta}\right) + \underline{\mu}(1-\nu)\left[C_q\left(\overline{q}, \overline{\theta}\right) - C_q\left(\overline{q}, \underline{\theta}\right)\right] - \nu C_q\left(\overline{q}, \underline{\theta}\right) \\
&= (1-\nu) C_q\left(\overline{q}, \overline{\theta}\right) + \underline{\mu}(1-\nu)\left[C_q\left(\overline{q}, \overline{\theta}\right) - C_q\left(\overline{q}, \underline{\theta}\right)\right] \\
&\quad + \nu\left[C_q\left(\overline{q}, \overline{\theta}\right) - C_q\left(\overline{q}, \underline{\theta}\right)\right]
\end{aligned}$$

$$\Leftrightarrow S'(\overline{q}) = C_q\left(\overline{q}, \overline{\theta}\right) + \underbrace{\left[\underline{\mu} + \frac{\nu}{1-\nu}\right]}_{=\frac{\nu\overline{\mu}}{1-\nu},\ \text{where } \overline{\mu}>0} \underbrace{\int_{\underline{\theta}}^{\overline{\theta}} C_{q\theta}\left(\overline{q}, \theta\right) d\theta}_{>0}.$$

$$\boxed{\Rightarrow \overline{q}^{SB} < \overline{q}^{FB}.}$$

- That is, the bad type's second-best quantity is lower than the first-best quantity.
- Now rewrite (3) using (6):

$$\begin{aligned}
\nu S'\left(\underline{q}\right) &= \overline{\mu}\nu C_q\left(\underline{q}, \underline{\theta}\right) - \underline{\mu}(1-\nu) C_q\left(\underline{q}, \overline{\theta}\right) \\
&= \nu C_q\left(\underline{q}, \underline{\theta}\right) - \underline{\mu}(1-\nu)\left[C_q\left(\underline{q}, \overline{\theta}\right) - C_q\left(\underline{q}, \underline{\theta}\right)\right] \Leftrightarrow
\end{aligned}$$

$$S'\left(\underline{q}\right) = C_q\left(\underline{q}, \underline{\theta}\right) - \frac{\underline{\mu}(1-\nu)}{\nu} \underbrace{\int_{\underline{\theta}}^{\overline{\theta}} C_{q\theta}\left(\underline{q}, \theta\right) d\theta}_{>0}.$$

$$\boxed{\Rightarrow \underline{q}^{SB} \geq \underline{q}^{FB} \qquad \text{with eq. iff } \underline{\mu} = 0.}$$

- That is, the good type's second-best quantity is either the same as the first-best quantity or larger, depending on whether IG-good binds or not.
- As far as I am aware, it is not possible to determine in general whether $\underline{\mu} = 0$ or $\underline{\mu} > 0$ without specifying more specific functional forms.
  - Still, even without that info we can rule out that $\underline{q}^{SB} < \underline{q}^{FB}$.

- Moreover, below I offer a non-formalized argument for why we must have $\underline{\mu} = 0$ (and therefore $\underline{q}^{SB} = \underline{q}^{FB}$) for values of γ that are strictly positive but close enough to zero.

- The IG constraints can be written as:

$$(1-\nu)\left[\bar{t} - \underline{t} + C\left(\underline{q}, \bar{\theta}\right) - C\left(\bar{q}, \bar{\theta}\right)\right] \geq \gamma, \qquad \text{(IG-good)}$$

$$\nu\left[\underline{t} - \bar{t} + C\left(\bar{q}, \underline{\theta}\right) - C\left(\underline{q}, \underline{\theta}\right)\right] \geq \gamma. \qquad \text{(IG-bad)}$$

- By inspecting these inequalities we see that in the limit as $\gamma \to 0$:
  - IG-good is satisfied iff IC-bad is satisfied.
  - IG-bad is satisfied iff IC-good is satisfied.
- Therefore, in the limit as $\gamma \to 0$ we are back to the standard model (i.e., the one without the IG constraints).
- We know that at the second-best optimum of the standard model IC-good binds and IC-bad is satisfied with a strict inequality.
  - But we know that in the limit as $\gamma \to 0$: IC-bad being satisfied with a strict inequality is equivalent to IG-good being satisfied with a strict inequality.
- Therefore, in the limit as $\gamma \to 0$, IG–good is satisfied with a strict inequality.
- By continuity, IG-good must be satisfied with a strict inequality also for values of γ that are strictly positive but close enough to zero.
- Summing up:
  - The bad type's quantity is distorted downwards: $\bar{q}^{SB} < \bar{q}^{FB}$.
  - The good type's quantity is either equal to the first best level or distorted upwards: $\underline{q}^{SB} \geq \underline{q}^{FB}$. For values of γ (the cost of information gathering) that are strictly positive but small enough, the good type's quantity is not distorted, $\underline{q}^{SB} = \underline{q}^{FB}$.

Question 2

Consider the following moral hazard model with mean-variance preferences that we studied in the course. There is one (single) agent, A, and one principal, P. A chooses an effort level $e \in \Re_+$, thereby incurring the cost $c(e) = \frac{1}{2}e^2$. Given a choice of e, the output (i.e., A's performance) equals $q = e + z$, where z is an exogenous random term drawn from a normal distribution with mean zero and variance ν. It is assumed that P can observe q but not e. Moreover, neither P nor A can observe z. A's wage (i.e., the transfer from P to A) can only be contingent on the output q. It is restricted to be linear in q:

$$t = \alpha + \beta q = \alpha + \beta (e + z).$$

A is risk averse and has a CARA utility function: $U = -\exp[-r(t - c(e))]$, **where r (> 0) is the coefficient of absolute risk aversion. Therefore A's expected utility is**

$$EU = -\int_{-\infty}^{\infty} \exp[-r(t - c(e))] f(z)\, dz,$$

where $f(z)$ is the density of the normal distribution. P's objective function is

$$V = q - t = q - \alpha - \beta q = (1 - \beta)(e + z) - \alpha,$$

which in expected terms becomes $EV = (1 - \beta)e - \alpha$. It is also assumed that A's outside option utility is $\widehat{U} = -\exp\left[-r\widehat{t}\right]$, where $\widehat{t} > 0$. The timing of events is as follows.

1. **P chooses the contract parameters, α and β.**
2. **A accepts or rejects the contract and, if accepting, chooses an effort level.**
3. **The noise term z is realized and A and P get their payoffs.**

Answer the following questions:

a) **Solve for the β-parameter in the second-best optimal contract, denoted β^{SB} (you do not need to solve for α^{SB}, and you will not get any credit if you nevertheless do that). You should make use of the following (well-known) result:**

$$EU = -\exp\left[-r\left(\alpha + \beta e - \frac{1}{2}e^2 - \frac{1}{2}\nu r \beta^2\right)\right].$$

- P's chooses the parameters in the contract, α and β. In addition, P can effectively choose A's effort e, because P designs the incentives that A faces when deciding what effort to make. We can thus think of P as choosing α, β, and e in order to maximize his expected payoff, subject to A's incentive compatibility constraint. In addition, A's individual rationality constraint must be satisfied. P's problem:

$$\max_{\alpha,\beta,e}\left\{\overbrace{(1-\beta)\,e-\alpha}^{=EV}\right\}$$

subject to

$$\overbrace{-\int_{-\infty}^{\infty}\exp\left[-r\left(t-c\left(e\right)\right)\right]f\left(z\right)dz}^{=EU}\geq-\exp\left[-r\widehat{t}\right], \tag{IR}$$

$$e\in\arg\max_{e'}EU\left(e'\right). \tag{IC}$$

The IC constraint says that e indeed maximizes A's utility among all the e's that A could choose. The IR constraint says that A's expected utility if accepting the contract is at least as large as his utility from his outside option; this therefore ensures that A wants to participate.

- The IC constraint above is actually a whole set of infinitely many constraints. In order to reduce these to one single IC constraint, we can make use of the first-order approach, which means that we replace IC above with the first-order condition from A's maximization problem (for some arbitrary values of the contract parameters α and β). From the question we have that A's expected utility can be written as

$$EU=-\exp\left[-r\left(\alpha+\beta e-\frac{1}{2}e^{2}-\frac{1}{2}\nu r\beta^{2}\right)\right].$$

Maximizing EU is equivalent to maximizing a monotone transformation of this expression, so we can without loss of generality let A maximize

$$\widetilde{EU}=\alpha+\beta e-\frac{1}{2}e^{2}-\frac{1}{2}\nu r\beta^{2}. \tag{7}$$

- We have

$$\frac{\partial\widetilde{EU}}{\partial e}=\beta-e=0$$

Therefore A's optimal effort level is

$$e=\beta. \tag{8}$$

- We can write the IR constraint as

$$\begin{aligned}
-\int_{-\infty}^{\infty} \exp\left[-r\left(t-c\left(e\right)\right)\right] f\left(z\right) dz &\geq -\exp\left[-r\widehat{t}\right] \Leftrightarrow \\
-\exp\left[-r\left(\alpha+\beta e-\frac{1}{2}e^2-\frac{1}{2}\nu r\beta^2\right)\right] &\geq -\exp\left[-r\widehat{t}\right] \Leftrightarrow \\
\exp\left[-r\left(\alpha+\beta e-\frac{1}{2}e^2-\frac{1}{2}\nu r\beta^2\right)\right] &\leq \exp\left[-r\widehat{t}\right] \Leftrightarrow \\
-r\left(\alpha+\beta e-\frac{1}{2}e^2-\frac{1}{2}\nu r\beta^2\right) &\leq -r\widehat{t} \Leftrightarrow \\
\alpha+\beta e-\frac{1}{2}e^2-\frac{1}{2}\nu r\beta^2 &\geq \widehat{t} \Leftrightarrow \\
\alpha &\geq \widehat{t}-\beta e+\frac{1}{2}e^2+\frac{1}{2}\nu r\beta^2
\end{aligned}$$

  Plugging in (8) in this inequality, we obtain

$$\begin{aligned}
\alpha &\geq \widehat{t}-\beta^2+\frac{1}{2}\beta^2+\frac{1}{2}\nu r\beta^2 \\
&= \widehat{t}-\frac{1}{2}\left(1-\nu r\right)\beta^2.
\end{aligned}$$

  Plugging in (8) into P's objective function $EV=\left(1-\beta\right)e-\alpha$, we have

$$EV=\left(1-\beta\right)\beta-\alpha.$$

- Using the above results, P's problem becomes

$$\max_{\alpha,\beta}\left\{\left(1-\beta\right)\beta-\alpha\right\} \qquad \text{subject to}$$

$$\alpha\geq\widehat{t}-\frac{1}{2}\left(1-\nu r\right)\beta^2. \tag{IR}$$

- It is clear that IR must bind, as the objective is decreasing in α and the constraint is tightened as α is lowered (thus P wants to lower α until the constraint says stop). We thus have $\alpha=\widehat{t}-\frac{1}{2}\left(1-\nu r\right)\beta^2$. Plugging this value of α into the objective yields the following unconstrained problem:

$$\boxed{\max_{\beta}\left\{\beta-\frac{1}{2}\left(1+\nu r\right)\beta^2-\widehat{t}\right\},}$$

  with the first-order condition

$$1-\left(1+\nu r\right)\beta=0\Rightarrow\beta^{SB}=\frac{1}{1+\nu r}.$$

b) **[You are encouraged to attempt parts b), c) and d) even if you have not been able to answer parts a).] Does the agent get any rents at the second-best optimum? Do not only answer yes or no, but also explain how you can tell. [PLEASE TURN THE PAGE!]**

- No, he does not get any rents at the second-best optimum. "Rents" are defined as any payoff from accepting the contract that exceeds the outside option payoff. However, we saw under a) that the IR constraint binds at the optimum, which means that A does not get any rents.

c) **The first-best values of the effort level and the β-parameter equal $e^{FB} = 1$ and $\beta^{FB} = 0$, respectively. How do these values relate to the corresponding second-best values? In particular, is there under- or overprovision of effort at the second-best optimum?**

- We have from the above analysis that $\beta^{SB} = e^{SB} = \frac{1}{1+\nu r}$. We see that there is underprovision of effort (as $e^{SB} < e^{FB}$). We also see that the beta-parameter is too high relative to the first best level ($\beta^{SB} > \beta^{FB}$).

d) **Consider the limit case where $r \to 0$. Explain what happens to the relationship between the second-best and the first-best effort levels. Also explain the intuition for this result.**

- In the limit where $r \to 0$, A is risk neutral. We see from above that in that limit, $e^{SB} = 1$. That is, the second-best effort level coincides with the first-best level: there is no inefficiency in spite of the fact that there asymmetric information. The reason why this can occur is that when risk neutral, A doesn't mind bearing risk. Therefore P can incentivize A very strongly, so that indeed $\beta^{SB} \to 1$ as $r \to 0$: A's compensation depends fully on the stochastic variation, so he makes the same decision as P would have made if he had been in A's job.
- The intuition is the same as we have discussed in other parts of the course, for example in the 2x2 moral hazard model with a risk neutral agent who is not protected by limited liability. There we explained the intuition as follows:
  - The economic meaning of the fact that A is risk neutral is that he cares only about whether his payment t is large enough *on average*. Hence, P can, without violating the participation constraint, incentivize A by giving him a negative payment (in practice a penalty) in case of a low output. More generally, P can achieve the first-best outcome by making A the residual claimant:
    - Then A effectively buys the right to receive any returns: "the firm is sold to the agent".
    - Thereby, the effort level is chosen by the same individual who bears the consequences of the choice.
    - In this situation A makes the same effort choice as P would have made.